Probability

Finite Math

9 May 2019

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Theorem (Probability of the Union of Two Events)

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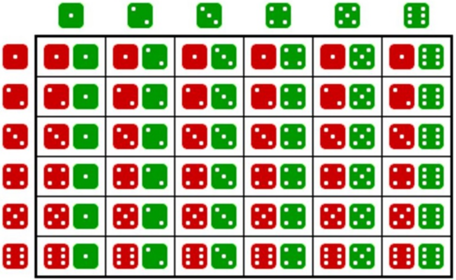
Remark

Note the similarity to the formula for the addition principle:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Example



Example

Suppose that two fair dice are rolled.

(a) *What is the probability that a sum of 7 or 11 turns up?*

(b) *What is the probability that both dice turn up the same or that a sum less than 5 turns up?*

Example

Example

What is the probability that a number selected at random from the first 500 *positive integers is:*

(a) *divisible by* 3 *or* 4 *?*

(b) *divisible by* 4 *or* 6 *?*

Complements

Let $S = \{e_1, e_2, ..., e_n\}$ be a sample space and let E be some event. Then the set E' is also an event and since $E \cap E' = \varnothing$ and $E \cup E' = S$, then we have

$$P(S) = P(E \cup E') = P(E) + P(E') = 1.$$

So it follows that

$$P(E) = 1 - P(E') \qquad \textit{and} \qquad P(E') = 1 - P(E).$$

If E is an event, then E' is "the event that E *does not* happens."
As a simple example, if E is the event that it snows outside and $P(E) = .35$, then E' is the event that it does not snow and $P(E') = .65 = 1 - .35$.

Example

Example

A shipment of 45 *precision parts, including* 9 *that are defective, is sent to an assembly plant. The quality control division selects* 10 *at random for testing and rejects the entire shipment if* 1 *or more in the sample are found to be defective. What is the probability that the shipment will be rejected?*

Now You Try It!

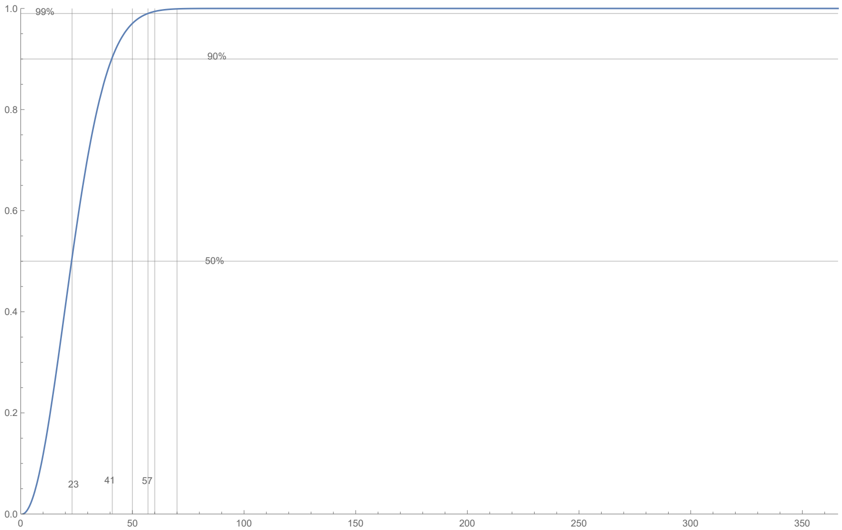
Example

A shipment of 40 *precision parts, including* 8 *that are defective, is sent to an assembly plant. The quality control division selects* 10 *at random for testing and rejects the entire shipment if* 1 *or more in the sample are found to be defective. What is the probability that the shipment will be rejected?*

Birthday Paradox!

Example

Let's assume there are 365 days in a year (sorry anyone born on February 29th). In a group of n people, what is the probability that at least 2 people have the same birthday? What value of n is required for the probability to be at least 50%? 90%? 99%?


1.0
0.8
0.6
0.4
0.2
0.0
99%
90%
50%
23
41
57
0
50
100
150
200
250
300
350

Empirical Application

Example

From a survey involving 1,000 *university students, a market research company found that* 750 *students owned laptops,* 450 *owned cars, and* 350 *owned cars and laptops. If a university student is selected at random, what is the (empirical) probability that:*

(a) *The student owns either a car or a laptop?*

(b) *The student owns neither a car nor a laptop?*

Now You Try It!

Example

From a survey of 1,000 people in Springfield, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a person from Springfield is selected at random, what is the (empirical) probability that:

(a) *They have tried the diet cola or the regular cola?*

(b) *They have tried one of the colas, but not both?*

New Idea

In the experiments we were doing earlier that involved doing things multiple times, the outcome of the first thing did not influence the later outcomes. Suppose instead our experiment were something like pulling colored beads out of a pot where each time we pull one out, we do not put it back in. Then, each time we pull out a bead, the probabilities for pulling out different colors on subsequent draws changes. This is the notion of *conditional probability*, denoted by $P(A|B)$ "the probability of A given B."

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Example

Suppose we roll a single fair die (6 sided), but cannot see the outcome. What is the probability that we rolled a prime number given that someone has told us that we rolled an odd number?

Conditional Probability

Definition (Conditional Probability)

For events A and B in an arbitrary sample space S, we define the conditional probability of A given B by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where $P(B) \neq 0$.

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Example

Suppose we roll two fair dice (6 sided). What is the probability that:

(a) *The sum is less than* 6 *given that the sum is even.*

(b) *The sum is* 10 *given that the roll is doubles.*

(c) *The sum is even given that the sum is less than* 6.

(d) *The sum is odd given that at least one die is a six.*

Application of Conditional Probability

Example

Suppose that city records produced the following probability data on a driver being in an accident on the last day of a Memorial Day weekend:

	Accident (A)	**No Accident** (A')	**Totals**
Rain (R)	.025	.335	.360
No Rain (R')	.015	.625	.640
Totals	.040	.960	1.000

(a) *Find the probability of an accident, rain or no rain.*

(b) *Find the probability of rain, accident or no accident.*

(c) *Find the probability of an accident and rain.*

(d) *Find the probability of an accident, given rain.*